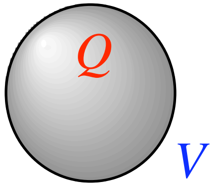
静電容量　(Capacitance)

空間に孤立した導体に電荷Ｑが帯電

１個

導体の電位をＶとするとき，**静電容量の定義は**



導体の形は任意

$$C = \frac{Q}{V} \left[\frac{\mathrm{C}}{\mathrm{V}}\right]$$

$$C = \frac{Q}{V} \left[\mathrm{F}\right]$$

単位

$$1\left[\frac{\mathrm{C}}{\mathrm{V}}\right] = 1\left[\mathrm{F}\right]$$ Farad

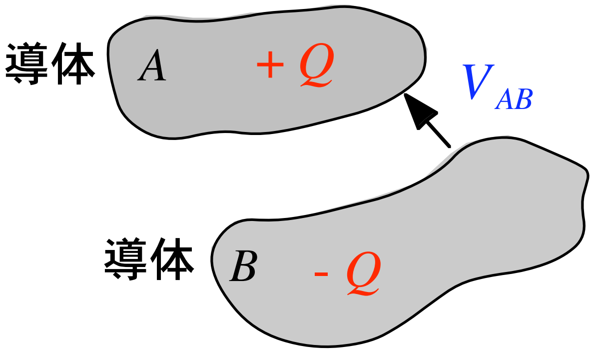
$$1\left[\mu\mathrm{F}\right] = 10^{-6}\left[\mathrm{F}\right]$$

$$1\left[\mathrm{pF}\right] = 10^{-12}\left[\mathrm{F}\right]$$

２つの導体A, Bがあるとき，導体Aに＋Ｑ，導体Bに－Ｑの電荷を与える。

２導体間の電位差がV_{AB}のとき，**静電容量の定義は**

２個



導体の形は任意

$$C = \frac{Q}{V_{AB}} \left[\mathrm{F}\right]$$

導体では，どの場所でも電位は同じなので，AB間の電位差は一定

電荷を蓄えるために作ったデバイス

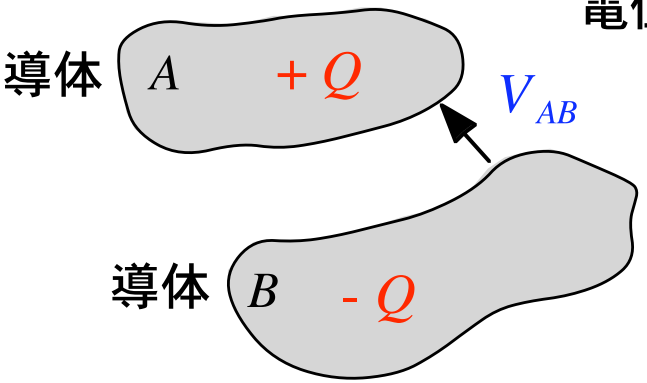
→ Capacitor (Condensor)

定義式をしっかりと

$$C = \frac{Q}{V_{AB}} = \frac{\int_v \rho_v \, dv}{-\int_B^A \boldsymbol{E} \cdot d\boldsymbol{l}} = \frac{\oint_S \varepsilon \boldsymbol{E} \cdot d\boldsymbol{S}}{-\int_B^A \boldsymbol{E} \cdot d\boldsymbol{l}}$$

導体の形状と誘電率によって決まる

電界そのものには依存しない



導体の形は任意

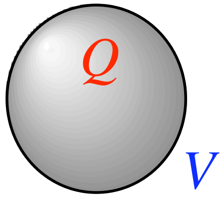
電位差 $V_{AB} = V_A - V_B = -\int_B^A \boldsymbol{E} \cdot d\boldsymbol{l} = -\int_-^+ \boldsymbol{E} \cdot d\boldsymbol{l}$

$B \Rightarrow \infty \qquad V_B = -\int_\infty^\infty \boldsymbol{E} \cdot d\boldsymbol{l} = 0$

孤立導体 $V_{A\infty} = V_A = -\int_\infty^A \boldsymbol{E} \cdot d\boldsymbol{l}$

例　　1 個の導体球の静電容量

半径a, 全電荷Q の導体球では，電位 V は

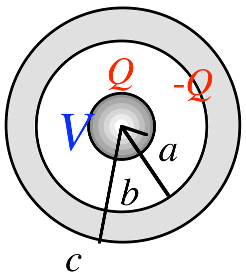


$$V = \frac{Q}{4\pi\varepsilon_0 a} \qquad C = \frac{Q}{V} = 4\pi\varepsilon_0 a \ \text{[F]}$$

半径　$a = 9$ cm　$C = 4\pi\varepsilon_0 a = \dfrac{9\times10^{-2}}{9\times10^{9}} = 10^{-11}$ [F] = 10 [pF]

地球　$C = \dfrac{\frac{2}{\pi}\times10^{7}}{9\times10^{9}} = 7.07\times10^{-4}$ [F] = 707 [μF]

同心球では



$a < r < b$　で電界は　$\boldsymbol{E} = E_r \boldsymbol{a}_r = \dfrac{Q}{4\pi\varepsilon_0 r^2}\boldsymbol{a}_r$

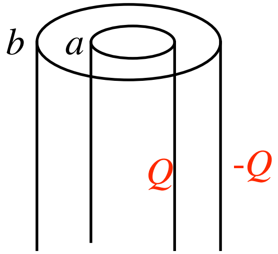
電位差

$$V_{AB} = -\int_b^a \boldsymbol{E}\cdot d\boldsymbol{l} = -\int_b^a \frac{Q}{4\pi\varepsilon_0 r^2}\boldsymbol{a}_r \cdot dr\,\boldsymbol{a}_r = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$C = \frac{4\pi\varepsilon_0}{\left(\frac{1}{a} - \frac{1}{b}\right)} \ \text{[F]}$$

なお　$b \to \infty$　$C \Rightarrow 4\pi\varepsilon_0 a$

同心円筒



単位長あたり

$a < \rho < b$ で電界はガウスの法則より $\boldsymbol{E} = \dfrac{Q}{2\pi \varepsilon_0 \rho} \boldsymbol{a}_\rho$

電位差

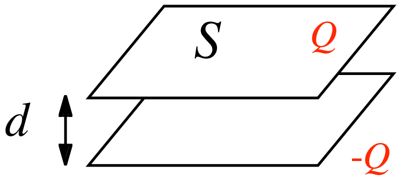
$$V_{AB} = -\int_b^a \boldsymbol{E} \cdot d\boldsymbol{l} = -\int_b^a \frac{Q}{2\pi \varepsilon_0 \rho} \boldsymbol{a}_\rho \cdot d\rho\, \boldsymbol{a}_\rho = \frac{Q}{2\pi \varepsilon_0} \ln \frac{b}{a}$$

$$C = \frac{Q}{V} = \frac{2\pi \varepsilon_0}{\ln \frac{b}{a}} \ [\mathrm{F}]$$

例えば $a = 0.5$ mm , $b = 4$ mm

$$C = \frac{2\pi \varepsilon_0}{\ln \frac{b}{a}} = \frac{1}{2 \times 9 \times 10^{9} \times \ln(4/0.5)} = 2.7 \times 10^{-11} \ [\mathrm{F}] = 27 \ [\mathrm{pF}]$$

平行平板



$0 < z < d$ で電界は $\boldsymbol{E} = -\dfrac{\rho_s}{\varepsilon_0} \boldsymbol{a}_z = -\dfrac{Q}{\varepsilon_0 S} \boldsymbol{a}_z$

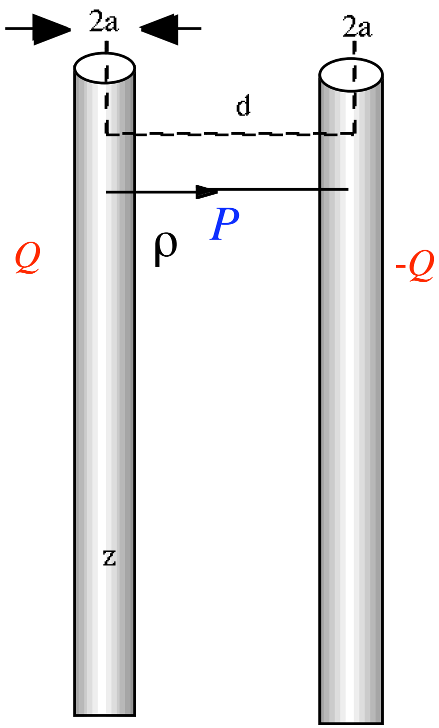
$$V_{AB} = -\int_B^A \boldsymbol{E} \cdot d\boldsymbol{l} = -\int_0^d -\frac{Q}{\varepsilon_0 S} \boldsymbol{a}_z \cdot dz\, \boldsymbol{a}_z = \frac{Q\, d}{\varepsilon_0 S}$$

$$C = \frac{Q}{V} = \frac{\varepsilon_0 S}{d} \ [\mathrm{F}]$$

例えば $S = 10$ cm ^{2} , $d = 1$ mm

$$C = \frac{\varepsilon_0 S}{d} = \frac{8.854 \times 10^{-12} \times 10 \times 10^{-4}}{1 \times 10^{-3}} = 8.854 \times 10^{-12} \ [\mathrm{F}] = 8.854 \ [\mathrm{pF}]$$

無限長平行導線間の静電容量



点Pでの電界は，２つの電荷からの合成

単位長あたり Q の線電荷があると仮定すると

$$E_\rho = E_\rho^+ + E_\rho^- = \frac{Q}{2\pi\varepsilon_0}\left(\frac{1}{\rho} + \frac{1}{d-\rho}\right)$$

$$V_{AB} = -\int \boldsymbol{E}\cdot d\boldsymbol{l} = -\frac{Q}{2\pi\,\varepsilon_0}\int_{d-a}^{a}\left(\frac{1}{\rho} + \frac{1}{d-\rho}\right)\boldsymbol{a}_\rho \cdot d\rho\,\boldsymbol{a}_\rho$$

$$= \frac{Q}{2\,\pi\,\varepsilon_0}\left\{\left[\,\ln\rho\,\right]_a^{d-a} + \left[\,\ln(d-\rho)\,\right]_a^{d-a}\right\} = \frac{Q}{\pi\,\varepsilon_0}\ln\frac{d-a}{a}$$

$$C = \frac{Q}{V} = \frac{\pi\,\varepsilon_0}{\ln\dfrac{d-a}{a}}\ \text{[F/m]}$$ 単位長あたり

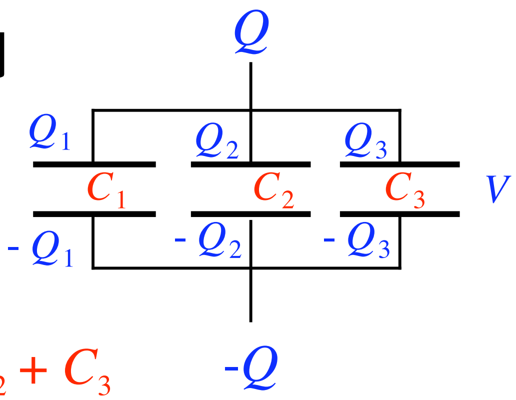
もし $a = 1$ mm , $b = 1$ m なら

$$C = \frac{\pi\,\varepsilon_0}{\ln\dfrac{d-a}{a}} = 4\times 10^{-12}\ \text{[F]} = 4\ \ \text{[pF]}$$

$d >> a \Longrightarrow C = \dfrac{\pi\,\varepsilon_0}{\ln\dfrac{d}{a}}$ [F/m]

並列接続と直列接続 $C = \dfrac{Q}{V}$

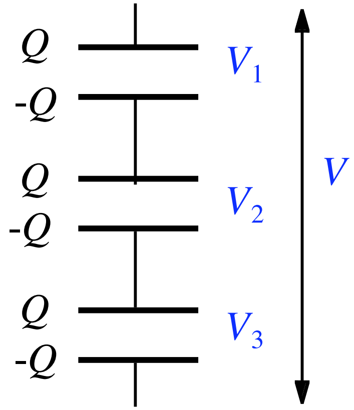
並列



$V = constant$ $\quad Q = Q_1 + Q_2 + Q_3$ に分割

$$C = \frac{Q}{V} = \frac{Q_1 + Q_2 + Q_3}{V} = \frac{Q_1}{V} + \frac{Q_2}{V} + \frac{Q_3}{V} = C_1 + C_2 + C_3$$

直列

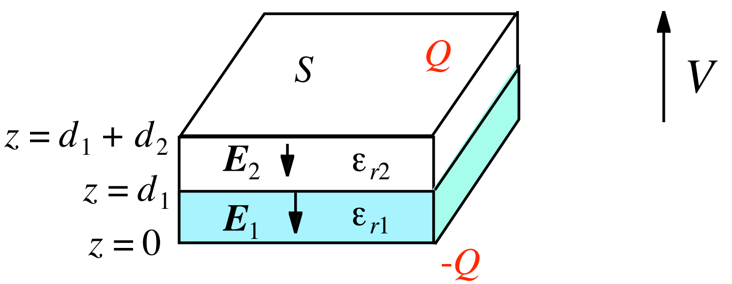


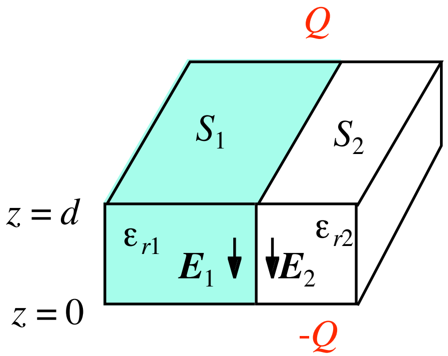
$Q = constant$ $\quad V = V_1 + V_2 + V_3$ に分割

$$C = \frac{Q}{V} = \frac{Q}{V_1 + V_2 + V_3}$$

$$\frac{1}{C} = \frac{V}{Q} = \frac{V_1 + V_2 + V_3}{Q} = \frac{V_1}{Q} + \frac{V_2}{Q} + \frac{V_3}{Q} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

誘電体と静電容量





$D_{1n}=D_{2n}$　　境界条件　　$E_{1t}=E_{2t}$

電荷は　$Q=D\,S$

$$E_1=\frac{D_1}{\varepsilon_0\,\varepsilon_{r1}}=\frac{Q}{\varepsilon_0\,\varepsilon_{r1}\,S}\qquad E_2=\frac{D_2}{\varepsilon_0\,\varepsilon_{r2}}=\frac{Q}{\varepsilon_0\,\varepsilon_{r2}\,S}$$

電圧は

$$V=-\int_0^{d_1+d_2}\boldsymbol{E}\cdot dl$$

$$V=E_2\,d_2+E_1\,d_1=\frac{d_1\,Q}{\varepsilon_0\,\varepsilon_{r1}\,S}+\frac{d_2\,Q}{\varepsilon_0\,\varepsilon_{r2}\,S}$$

$$C=\frac{Q}{V}=\frac{\varepsilon_0\,S}{\dfrac{d_1}{\varepsilon_{r1}}+\dfrac{d_2}{\varepsilon_{r2}}}$$

直列

電界は

$$E_1=E_2=\frac{V}{d}$$

$$D_1=\varepsilon_0\,\varepsilon_{r1}\,E_1\qquad D_2=\varepsilon_0\,\varepsilon_{r2}\,E_2$$

電極における面電荷密度は

$$\rho_{s1}=D_1=\varepsilon_0\,\varepsilon_{r1}\,\frac{V}{d}\qquad \rho_{s2}=D_2=\varepsilon_0\,\varepsilon_{r2}\,\frac{V}{d}$$

電荷は　$Q=\rho_{s1}\,S_1+\rho_{s2}\,S_2$

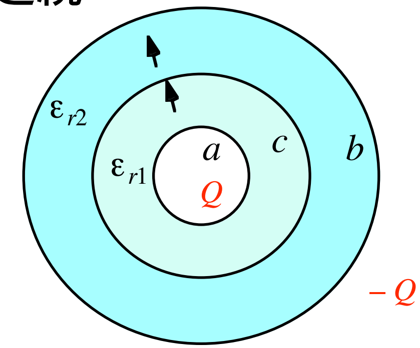
$$C=\frac{Q}{V}=\frac{\varepsilon_0\left(\varepsilon_{r1}\,S_1+\varepsilon_{r2}\,S_2\right)}{d}$$

並列

$$D_{1n} = D_{2n} \qquad \text{境界条件} \qquad E_{1t} = E_{2t}$$

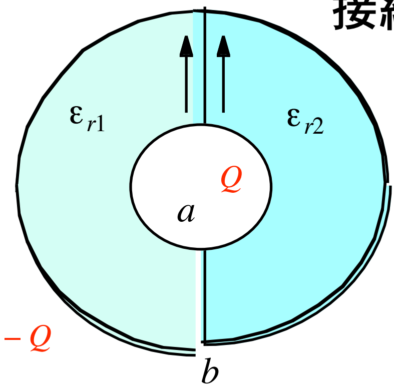
法線成分が連続

$$D_r = \frac{Q}{4\pi r^2}$$



球

接線成分が連続



電界は等しい

ガウスの法則

電界は

$$E_{r1} = \frac{Q}{4\pi\varepsilon_0\,\varepsilon_{r1}\,r^2} \qquad E_{r2} = \frac{Q}{4\pi\varepsilon_0\,\varepsilon_{r2}\,r^2}$$

$$V = -\int_b^a E_r\,dr$$

$$= \frac{Q}{4\pi\varepsilon_0\,\varepsilon_{r1}}\left(\frac{1}{a} - \frac{1}{c}\right) + \frac{Q}{4\pi\varepsilon_0\,\varepsilon_{r2}}\left(\frac{1}{c} - \frac{1}{b}\right)$$

$$C = \frac{4\,\pi\,\varepsilon_0}{\left(\frac{1}{a} - \frac{1}{c}\right)\frac{1}{\varepsilon_{r1}} + \left(\frac{1}{c} - \frac{1}{b}\right)\frac{1}{\varepsilon_{r2}}}$$

電荷は　$Q = D_{s1}\,2\pi r^2 + D_{s2}\,2\pi r^2$

$$= \varepsilon_0\varepsilon_{r1}\,E_r\,2\pi r^2 + \varepsilon_0\varepsilon_{r2}\,E_r\,2\pi r^2$$

$$E_r = \frac{Q}{2\pi\,\varepsilon_0\left(\varepsilon_{r1} + \varepsilon_{r2}\right)r^2}$$

$$V = -\int_b^a E_r\,dr = \frac{Q}{2\pi\,\varepsilon_0\left(\varepsilon_{r1} + \varepsilon_{r2}\right)}\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$C = \frac{2\,\pi\,\varepsilon_0\,(\varepsilon_{r1} + \varepsilon_{r2})}{\left(\frac{1}{a} - \frac{1}{b}\right)}$$